Revisiting battery modeling using the energy power supply concept

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***Abstract*—Using the energy power supply concept, this paper proposes a DC voltage source with limited energy storage capacity, which models: the energy storage capacity of the batteries, the involved energy transfer process and the constant voltage output behavior when the battery has storage energy. Further, this circuit source is used for understanding the combined electrical circuit-based battery model, which is commonly used for designing of: portable electronic devices, hybrid electric vehicles and smart grid systems.**

Keywords*—Battery modeling, electrical circuit-based battery model, Energy, Power, DC source.

I. Introduction

The energy storage devices have a wide application span. Recently, the battery technologies have been receiving huge attention because three commercial applications, whose push the batteries to its limits [1]. These are: portable electronic devices (PEDs), hybrid electric vehicles and smart grid systems. In the design of PEDs, the trade-off between functionality (i.e. dissipated power) and portability (i.e. battery running time, weight, size) [2] must be faced. Typically, the device portability is associated to the battery because it is the least likely element to change in the development cycle of the product [3]. Without battery models in hand, circuit designers cannot predict nor optimize either battery runtime or circuit performance. In order to integrate the batteries as energy storage devices into the smart grid, it is important to evaluate the utilization of them in the system [4], in terms of cost (e.g. maintenance), Lifetime (measured by the charge discharge cycles and calendar life of the battery), power delivery (i.e. chargedischarge rate, energy storage level, ramp rate, and chargedischarge efficiency), environmental impact and safety. This evaluation is much cheaper if it is made by electrical simulation instead of prototyping [5]. However, the simulation results are dependent on the accuracy of battery model. The electric vehicle industry use large battery packs in demanding applications that involve high power charge and discharge rates, with high service life [6]. An accurate and efficient battery model is vital to increase the performance of the driver and its battery, and to inform its development to the control systems [4].

Battery modeling is a broad and complicated field, with no single model capable of meeting the requirements for all applications. The battery models may be classified as physical models, analytical models, and circuit-based models [1], [6]. The physical models are mainly used to optimize the physical design aspects of batteries, they capture the fundamental mechanisms of power generation. The analytical models adopt empirical equations (e.g. math fitting of empirical data) to predict system-level behaviors, such as battery runtime, efficiency, or capacity. The circuit-based models use a combination of circuit elements (i.e. voltage sources, resistors, and capacitors) to predict the I–V information of the battery. For electrical engineers, this circuit models are more intuitive, useful, and easy to handle, especially when they can be used in circuit simulators (i.e. compatible with spice models) for co-simulation with other circuits.

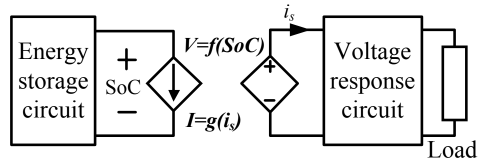


Fig. 1. Combined electrical–circuit based model.

This paper focus on the circuit-based models proposed in [7], which was referenced as the combined electrical circuit-based model (CECBM) by [1], [6]. This model is commonly used for designing of: portable electronic devices, hybrid electric vehicles and smart grid system, because it is capable to predict accurately the DC voltage response, the runtime and the transient. Further, it was modified by [8]–[11] for capturing some nonlinear behaviors (e.g. the C-rate effect, state of health (SoH) of the battery). These family of models (illustrated in Fig. 1) are based on two separate circuits: energy storage circuit and voltage response circuit, these circuits are coupled by a nonlinear relation (i.e. f and g) that models the energy flow process. These models have been used without a comprehensive development about the inspiration of its topology, and the simple energy flow process that models. In this paper the electric Energy Power Supply (EPS) concept is presented, and a DC voltage source with limited energy storage capacity was proposed. Further, we use this component for understanding the behaviors captured by the CECBM.

II. Energy Power Supply (EPS) concepts

A. *Energy flow process*

An EPS is a system that supplies electrical energy to a load. The load energy (E_L) is the energy delivered by the EPS and consumed by the load during a time interval, this energy can be calculated as the integral with respect to time of the instantaneous power (P_L) at the terminals of the load, this energy is supplied by the combination of current (i_L) and voltage (v_L) therefore, the energy flow process can be described as:

$$P_L(t) = \frac{dE_L\left(t\right)}{dt} = v_L\left(t\right) i_L\left(t\right); \tag{1}$$

this process, without the voltage and current behaviors, may be describe by a hydraulic analogy. The proposed analogy is shown in the Fig. 2, and the variable relations brought in Table.I.

B. EPS taxonomy

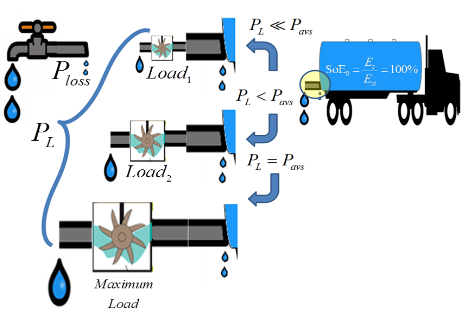
An ideal EPS must deliver an infinite energy at a required load power, in a small form factor. On the contrary, any real EPS delivers a limited energy at a limited power with a specific form factor. When an EPS is based on energy harvester, it supplies a high amount of energy (i.e. self-sustaining), but at a low power [12]. Additionally, this maximum power value (P_{avs}) can be extracted from the EPS only when the power transfer condition is satisfied. This type of source is referred as a power EPS. When the EPS is based on energy storage devices (e.g. batteries), it supplies a high power during a limited time, because it has a limited energy-storage-capacity (E_{st}). This type of source is referred as energy EPS. This taxonomy is summarized in Table II.

C. Ideal battery model

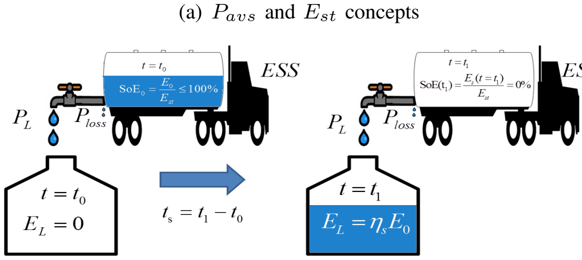
Assuming a time-invariant electric efficiency (η_S), the instantaneous stored energy in the EPS (E_S) and its generated power (P_S) are given by:

$$E_S(t) = \begin{cases} E_{st} & E_L(t) \leqslant -\eta_s(E_{st} - E_0) \\ E_0 - \frac{E_L(t)}{\eta_s} & E_0 - E_{st} < \frac{E_L(t)}{\eta_s} < E_0 \\ 0 & \eta_s E_0 \leqslant E_L(t) \end{cases}; \tag{2}$$

$$P_S(t) = -\frac{d}{dt}\left(\frac{E_L(t)}{\eta_s}\right) = -\frac{P_L(t)}{\eta_s}; \tag{3}$$



(a) P_{avs} and E_{st} concepts



(b) Runtime time concept

Fig. 2. Hydraulic analogy of a Loaded EPS.

TABLE I. Analogies between variables

Hydraulic variables	Unit	Electric variables	Unit
Volume of water	m^3	Electric energy (E)	J
Discharge	$\frac{m^3}{s}$	Electric power (P)	W
Volume of water stored in the tank at a particular instant.	m^3	Instantaneous electric energy stored in the EPS (E_s)	J
Tank volume	m^3	Energy storage capacity of the EPS (E_{st})	J
Tank filling percentage	$\frac{m^3}{m^3}$	State of energy (SoE) of the EPS	$\frac{J}{J}$
Tank initial filling percentage	$\frac{m^3}{m^3}$	initial state-of-energy (SoE_0)	$\frac{J}{J}$
Water tap coming-out discharge (WtD)	$\frac{m^3}{s}$	Power dissipated by the load (P_L)	W
Water tank coming-out discharge (WTD)	$\frac{m^3}{s}$	Generated power ($P_S = P_L + P_{loss}$)	W
WtD with totally open tap	$\frac{m^3}{s}$	Maximum available power of the EPS	W
Relationship between WtD and WTD	$\frac{m^3/s}{m^3/s}$	EPS efficiency $\eta = \frac{P_L}{P_S}$	$\frac{W}{W}$

where, E_0 is the initial stored energy in the energy EPS. Additionally, we define E_{st}, P_{avs} and the state of energy (SoE) as:

$$E_{st} = \lim_{t\to\infty} \int_0^t P_L\left(t\right) \cdot dt \quad , \text{ assuming } \lim_{t\to\infty} E_s(t) = 0; \tag{4}$$

$$P_{avs} = \max\left\{P_L\right\}; \tag{5}$$

$$SoE = \frac{E_s(t)}{E_{st}}; \tag{6}$$

the SoE is an indicator of the amount of energy stored in the EPS. For instance, when it is totally charged ($E_S = E_{st}$), this indicator is equal to one (SoE%= 100%). From (6) and (2) the SoE can be rewritten as:

$$SoE = \begin{cases} 1 & E_L(t) < -E_{st}\eta_s(1 - SoE_0) \\ SoE_0 - \frac{E_L(t)}{E_{st}\eta_s} & E_0 - E_{st} < \frac{E_L(t)}{\eta_s} < E_0 \\ 0 & \eta_s E_0 \leqslant E_L(t) \end{cases} \tag{7}$$

where, SoE_0 is the initial SoE of the EPS. Assuming a lossless EPS that supplies energy with a DC output voltage, i.e (8) and $\eta_S = 1$, the (7) can be rewritten as (9)

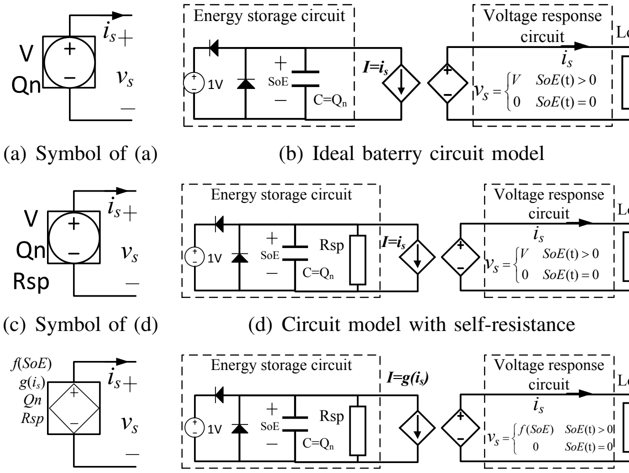
$$v_S\left(t\right) = \begin{cases} V & SoE(t) > 0 \\ 0 & SoE(t) = 0 \end{cases}; \tag{8}$$

$$SoE = \begin{cases} 1 & \frac{\int_0^t i_L(\tau)d\tau}{Q_n} \leqslant (SoE_0 - 1) \\ SoE_0 - \frac{\int_0^t i_L(\tau)d\tau}{Q_n} & \frac{SoE_0 - 1}{SoE_0} < \frac{\int_0^t i_L(\tau)d\tau}{Q_n SoE_0} < 1; \\ 0 & Q_n SoE_0 \leqslant \int_0^t i_L\left(\tau\right)d\tau \end{cases} \tag{9}$$

where i_L is the current supplied by the EPS to the load and Q_n is the charge storage capacity ($Q_n = E_{st}/V$) of the EPS. The electric behavior described by (9) can be represented by the circuit shown in Fig. 3(b). The symbol proposed for this component is shown in Fig. 3(a). This component was

TABLE II. EPS taxonomy

EPS Name	Pavs	Energy	Example
Ideal EPS	infinite	infinite	The sun
Real EPS	finite	finite	Electric distribution network
Power EPS	low	high	Solar Energy + PV cell
Energy EPS	high	low	AAA battery



(a) Symbol of (a) (b) Ideal baterry circuit model

(c) Symbol of (d) (d) Circuit model with self-resistance

(e) Symbol of (f) (f) Circuit model with nonlinear traking of SoE

Fig. 3. DC voltage source with limited energy storage capacity

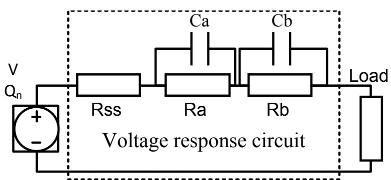


Fig. 4. Battery model proposed by [7] using the ideal battery model

referenced as a DC voltage source with limited energy storage capacity (i.e. voltage DC energy supply or ideal battery).

In conventional circuit theory the voltage source models can supply unlimited energy, therefore a direct battery model is unavailable, but using the circuit shown in the Fig.3(b) for modeling the battery, both its runtime (i.e. SoE as a function of the load current) and its I–V performance (i.e. output voltage as a function of the SoE) can be captured by the model. Adding three resistors and two capacitors (Fig. 4) to the ideal battery model, the following behaviors of the real battery were captured: limited output power, relaxation effect, and internal voltage drop when it is loaded. On the other hand, the self-discharge rate is proportional to SoE [13], therefore the energy storage circuit of the ideal battery must be modified for capturing the self-discharge effect, this is shown in Fig. 3. In the next section it is shown how these "ideal" models are modified for increasing its accuracy. The tracking of the SoE is modeled adding non linear relations (g and f) for describing the energy flow process as shown in the Fig. 3(f). Additionally, the component values of the voltage response circuit are modeled as non linear functions of: the SoE (i.e. more commonly used the SoC), the discharge current (I_L), the temperature (T), the charge and discharge time (t), and the number of discharge-charge cycles (N).

III. State of art of the CECBM

A brief list of the battery behaviors that may be captured by the CECBM is presented in Table IV. These models can be implemented in circuit simulators to predict both the battery runtime and I–V performance accurately. In [7] was proposed for the first time the CECBM, this model are illustrated using

TABLE IV. Significant electric battery behaviors

Battery behavior	Description
1) I–V performance (IVP)	I–V performance is the estimation of the battery output voltage for any load.
2) DC–IVP	DC I–V performance is the estimation of the steady state battery voltage variations
3) Open circuit voltage (V_{oc})	Is the voltage of battery during equilibrium state when the load is a open circuit
4) Internal resistance on DC–IVP	The internal impedance of the battery, when it increases the battery efficiency decreases and thermal stability is reduced.
5) N on DC–IVP	The DC–IVP changes with N, e.g. The Rss increase when N increase.
6) T on DC–IVP	The DC–IVP changes with the ΔT, e.g. the V_{oc} increase when N increase.
7) Transient response	Battery output voltage response when the load current is a rectangular current pulse.
8) State of Health (SoH) tracking	The SoH is a measurement of the ageing of the battery that reflects the general condition of a battery compared with a new battery. The accuracy estimation of the SoH by the model is referred as tracking the SoH.
9) SoC tracking	The SoC is more common than SoE in battery literature, but they describe the same energy flow process. The accuracy estimation of the SoC by the model is referred as tracking the SoC.
10) Capacity loss (i.e. storage energy loss)	Changes in the useable capacity (storage energy) can be either a result of irreversible capacity loss or reversible capacity change. Capacity fading depends on many stress factors such as temperature, C-rate, SoC and depth of discharge.
11) Capacity fading due to cycle number	Irreversible capacity loss (storage energy) as a result of cell ageing due to cycling.
12) Capacity fading due to time	Irreversible capacity loss (storage energy) as a result of cell ageing due to storage time.
13) capacity loss due to T	Reversible capacity change as a result of Δ T.
14) C-rate effect (rate capacity effect or current recovery effect)	Reversible capacity change as a result of the C-rate. The current recovery effect is the recovered capacity by discharging at a lower current.
15) Runtime prediction	The continuous period of time during which the battery operates as an energy source for its load (i.e. output voltage bigger than end-of-discharge voltage).
16) Self-discharge effect	The storage energy (called usable capacity) declines as storage time (self-discharge) increases.
17) AC response	I–V performance when the load current is an AC current.
18) Charge (Ch) and discharge (dCh)	Typically, the behaviors for discharging are not equal to charging behaviors.

TABLE V. Comparison of the proposed CECBMs

Behavior Table. IV	Fig. 3(b)	Fig. 3(d)	Fig. 4	2006 [7]	2009 [14]	2010 [8]	2012 [9]	2013 [10]	2011 [11]	2010 [15]
Item 4	no	yes	yes	yes	yes	yes	yes	yes	yes	yes
Item 5	no	no	no	no	yes	no	no	no	no	no
Item 6	no	no	no	no	yes	no	no	no	yes	no
Item 7	no	no	very lim	lim	lim	lim	lim	lim	lim	yes
Item 9	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Item 11	no	no	no	no	yes	no	no	no	no	no
Item 12	no	no	no	no	yes	no	no	no	no	no
Item 13	no	no	no	no	yes	no	no	no	yes	no
Item 14	no	no	no	no	no	yes	yes	yes	yes	no
Item 15	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Item 16	no	yes	yes	no	no	no	no	no	no	no
Item 17	no	no	no	no	no	no	no	no	no	no
Item 18	Ch&dCh	Ch&dCh	Ch&dCh	dCh	dCh	dCh	dCh	dCh	Ch&dCh	dCh

TABLE III. COMPARISON OF THE CIRCUIT VALUES DEPENDENCES ON THE ANALYZED MODELS

Model param.	Fig. 3(b)	Fig. 3(d)	Fig. 4	2006 [7]	2009 [14]	2010 [8]	2012 [9]	2013 [10]	2011 [11]
Qn	Nominal	Nominal	Nominal	Nominal	Nominal	Nominal	Nominal	Nominal	Nominal
Rsp	infinite	Nominal	Nominal	infinite	infinite	infinite	infinite	infinite	infinite
Rss	0	0	Nominal	$h(SoC)$	$h(SoC)$	$h(SoC)$	$h(SoC)$	$m(SoC, is)$	$q(SOC, T, abs(is), sgn(is))$
Ra, Rb	0	0	Fixed	$h(SoC)$	$h(SoC)$	$h(SoC)$	$h(SoC)$	$m(SoC, is)$	$q(SOC, T, abs(is), sgn(is))$
Ca, Cb	infinite	infinite	Fixed	$h(SoC)$	$h(SoC)$	$h(SoC)$	$h(SoC)$	$m(SoC, is)$	$q(SOC, T, abs(is), sgn(is))$
g	$g(is) = is$	$g(is) = is$	$g(is) = is$	$g(is) = is$	$g(is) = is/CF$	$l(is, t)$	$l(is, t)$	$l(abs(is))$	$n(abs(is), sgn(is), T)$
$\frac{f}{sgn(SoE)}$	$f(SoE) = V$	$f(SoE) = V$	$h(SoE)$	$h(SoE)$	$p(SoE, T)$	$h(SoE)$	$h(SoE)$	$h(SoE)$	$h(SoE)$

the ideal battery model in the Fig. 4, all component values of the voltage response circuit were proposed as a nonlinear functions of the SoC. In [15], the impact of the complexity of the voltage response circuit in the capture of transient response was analyzed. The authors demonstrate that with more than two RC circuits the battery model gives better accuracy. On the other hand, to improve the basic model with the capture of the C-rate effect, in [8] was used a more complex energy balance circuit based on a non linear tracking of the SoC, this circuit reflects the non linear decrease in the battery capacity (i.e. storage energy) when the discharge current increase. Other approach for modeling the C-rate effect was proposed in [9], they replace the energy balanced circuit for a math model based on the Kinetic Battery Model (KiBaM), these equations may be interpreted as nonlinear function of both the charging current and the time. In [14] was developed a battery model that captures the capacity fading effects, they propose a capacity correction factor as a non linear function of the storage time, temperature and cycle number, this factor may be also interpreted as a current correct factor. Recently, in [10] was proposed that the model parameters can be described as non linear functions of the SOC and the discharge current. Further, a more complex model was proposed in [11], they used empirical equations to describe the model parameters as nonlinear functions of: the current direction, the state of charge (SoC), the temperature, and C-rate. The brief comparison, illustrated in Table V, indicates the modeling road-map of the CECBM. From the designer point of view, the better model is the one that captures accurately the interest battery behaviors with minimum computational complexity, maximum flexibility, and stability. In the Table III is summarized the input variables used for extracting the circuit values of the models. The functions listed in this table ($h(x), p(x1, x2), l(x1, x2), m(x1, x2), q(x1, x2, x3, x4)$) are non linear equations that were fitted using the experimental data.

IV. CONCLUSIONS

The proposed DC voltage source with limited energy storage capacity was used for understanding the combined electrical circuit-based battery model. Further, it was presented a comprehensive state-of-the-art review of the progress that has been made on this battery model. Furthermore, a comprehensive development of the energy power supply concept and the energy flow process was presented and used for comparison of the analyzed models.

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