Revisiting the Power-Efficiency Trade-Off on a DC Voltage Source

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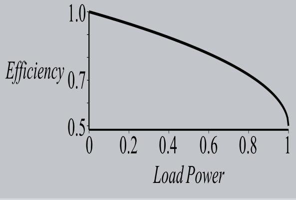
Power-efficiency trade-off I

Energy conversion processes are characterized by a trade-off between output power and efficiency.

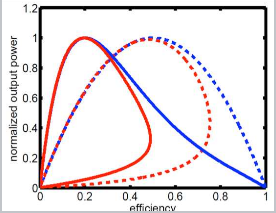
Earliest work about electric-mechanic conversion (1978)



Edison and his chief assistants researched the design techniques for DC electric generators, and determined that a generator with smaller internal resistance than its load is more efficient than a generator with internal resistance equal to its load [1].

Power-efficiency trade-off II

Nanoscale energy conversion Molecular motors (2016)



At the nanoscale new possibilities and phenomena arise. For example, chemical energy can be converted directly into useful work (molecular motors). This diagram shows the calculated output power versus efficiency of a molecular motor (dashed) near equilibrium and (solid) far from equilibrium for (blue) perfect coupling and (red) weak loss processes [2].

Power-efficiency trade-off III

PETO and Portable Electronic devices

- Most of the electronic devices must be portable and must be connected to the Internet [3]
- Typically, the device portability is associated to the energy technologies that has been adopted [4].
- The designer of PEDs must face a trade-off between functionality (i.e. dissipated power) and portability (i.e. battery running time, weight, size) [5].

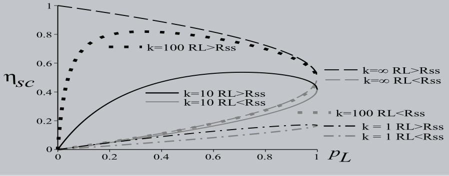
About this work

The problem

The designer of PEDs need to take into account the PETO in order to achieve the required specifications and, as a result, an optimum operating point (i.e. optimum load) could be found and then could be used as a constraint in the electronic design of the system.

The Proposed Solution

In this work an analytic expression is developed for quantifying the PETO on a DC Voltage Source with and without limited energy.

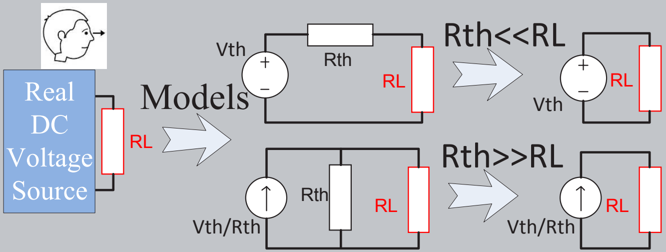


$$\eta_{s_c} = \frac{1 - \frac{p_L}{\xi}}{1 + \frac{\xi}{k \cdot p_L}}$$

Efficiency in a voltage source I

Modeling the DC source

A common technique for modeling a real voltage source is the use of the Norton or Thevenin equivalent circuit as its model.



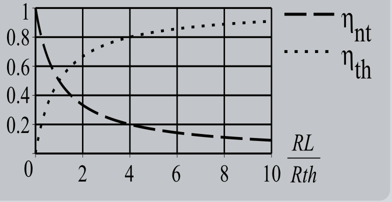
Efficiency in a voltage source II

Observation

The equivalent circuit predicts only the current and voltage performance of the source, from the load point of view.

Efficiency values predicted by both models

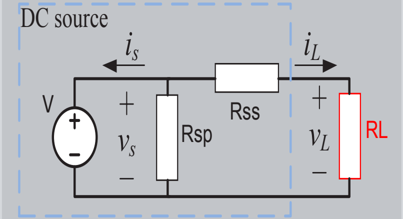
Without knowledge about the physical laws that govern the internal source behaviors, we cannot predict accurately its efficiency.



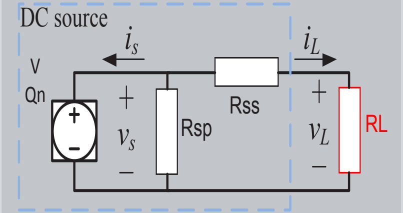
PETO for voltage source

The PETO is discussed for any energy source that can be modeled using the circuits shown:

With infinite energy



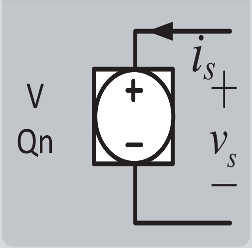
With finite energy



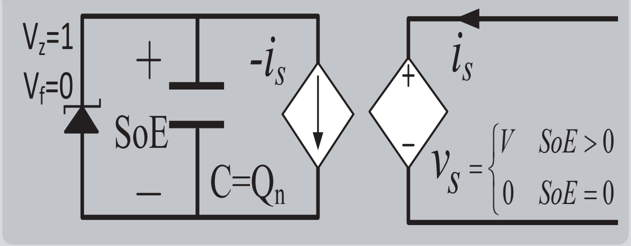
Ideal battery model

In [6] was proposed a battery model based on the circuit shown:

Symbol



Circuit model



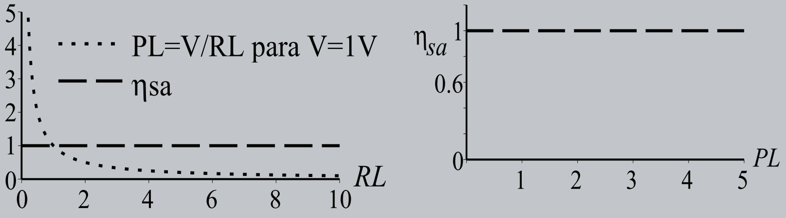
PETO $R_{ss} = 0, R_{sp} = \infty$

Efficiency and power expressions

For this circuit the efficiency is a load independent variable, therefore PETO does not occur.

$$\bar{P}_{L_a} = \frac{V^2}{RL}; \qquad (1)$$

$$\eta_{s_a} = 1. \qquad (2)$$



(a) Load power and efficiency vs load

(b) Efficiency vs power

PETO $R_{ss} \neq 0, R_{sp} = \infty$ I

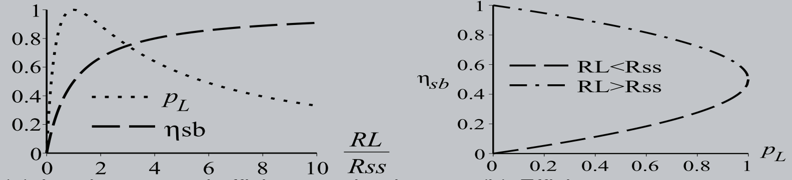
Efficiency and power expressions

$$\bar{P}_{L_b} = 4P_{avs}\frac{\frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}; \tag{3}$$

$$\eta_{s_b} = \frac{\eta_{s_a}}{1+\frac{Rss}{RL}}; \tag{4}$$

$$P_{avs} = \frac{V^2}{4 \cdot Rss}; \tag{5}$$

$$p_L = \frac{\bar{P}_{L_b}}{P_{avs}} = \frac{4 \cdot \frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}. \tag{6}$$



(a) Load power and efficiency vs load

(b) Efficiency vs power

PETO $R_{ss} \neq 0$, $R_{sp} = \infty$ II

Quantifying the PETO

- The better compromise between efficiency and power occurs in the load range where the PETO exists.
- The limits are: 100% of η and 100% of p_L.
- We cannot achieve these limits simultaneously. $(p_L = 0, \eta = 1)$ or $(p_L = 1, \eta = 0.5)$
- A good compromise of this PETO is $p_L = 0.75, \eta = 0.75$.

In all the operating points of this PETO, the relationship between the p_L and the η is described by:

$$\eta_{s_b} = 1 - \frac{p_L}{\xi};$$

where, ξ=2 $\left(1 + \sqrt{1 - p_L}\right)$, and $RL \geq Rss$.

PETO $R_{ss} \neq 0, R_{sp} \neq \infty$ I

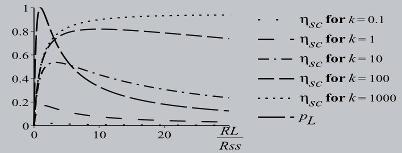
Efficiency and power expressions

$$\bar{P}_{L_c} = \bar{P}_{L_b} = 4P_{avs}\frac{\frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}; \qquad (7)$$

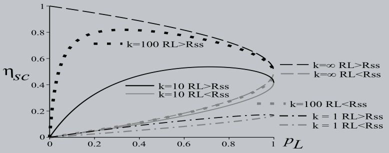
$$\eta_{sc} = \frac{\eta_{s_b}}{\left(\frac{\mathrm{Rss}}{\mathrm{Rsp}}\left(\frac{\mathrm{RL}}{\mathrm{Rss}}+1\right)+1\right)}; \qquad (8)$$

$$P_{avs} = \frac{V^2}{4\cdot Rss}; \qquad (9)$$

$$p_L = \frac{\bar{P}_{L_b}}{P_{avs}} = \frac{4\cdot\frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}. \qquad (10)$$



(a) Load power and efficiency vs load



(b) Efficiency vs power

PETO $R_{ss} \neq 0, R_{sp} \neq \infty$ II

Quantifying the PETO

- The PETO range is limited by the maximum η and minimum p_L point and the minimum η and maximum p_L point.
- Defining the source quality factor as $k = \frac{Rsp}{Rss}$:

$$\max\left\{\eta_{s_c}\right\} = \frac{k\sqrt{k+1}}{\left(1+k+\sqrt{k+1}\right)\left(k+\sqrt{k+1}\right)}$$

$$\max\left\{p_L\right\} = 1$$

- The extreme Load values are:

$$RL = RL_p = R_{ss}$$

$$RL = RL_\eta = Rss\sqrt{k+1}$$

- RL_p is the load value that maximizes p_L. RL_η is the load value that maximizes η

PETO $R_{ss} \neq 0$, $R_{sp} \neq \infty$ III

Quantifying the PETO

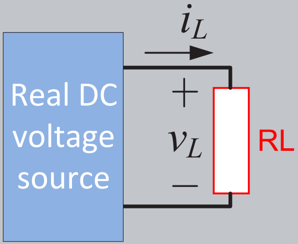
When RL is in the PETO range ($RL_p \leq RL \leq RL_\eta$), the load power can increase only if the efficiency decrease and vice versa. In all the operating points of this PETO, the relationship between the p_L and the η is described by:

$$\eta_{sc} = \frac{1 - \frac{p_L}{\xi}}{1 + \frac{\xi}{k \cdot p_L}}$$

where, $Rss\sqrt{k+1} \geq RL \geq Rss$ and ξ=2 $\left(1 + \sqrt{1 - p_L}\right)$. Having a high k value means that the power supply is good, i.e. both the self consumption and the internal conduction losses are low.

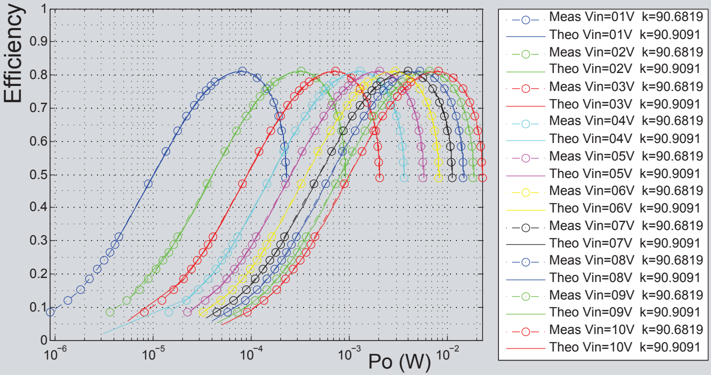
Experimental verification

Experimental Setup



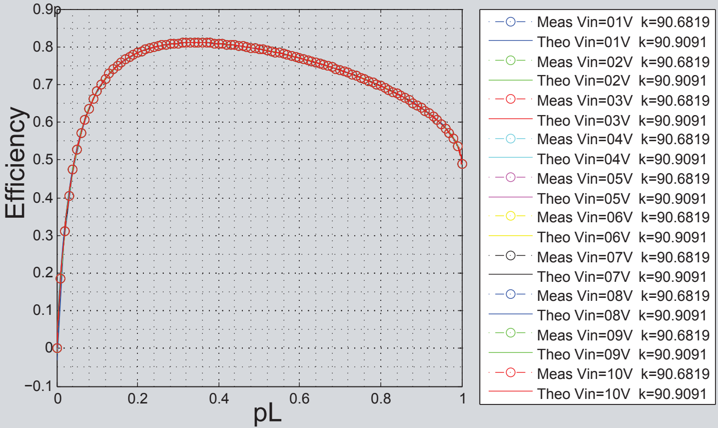
- The implemented system is a demonstrator of the validity of the analytic function rather than targeting a specific application.
- The real DC source ($k = 90$) was emulated using a power supply ($1\ \text{V} < V < 10\ \text{V}$) and two discrete resistors ($Rsp = 100\ \text{k}\Omega$, $Rss = 1.1\ \text{k}\Omega$), the electric load was an adjustable resistor ($100\text{m}\Omega < R_L < 10\ \text{M}\Omega$).
- Using this setup, both the resistive load and the input voltage were swept.

Experimental and theoretic results I



(a) Efficiency vs load power

Experimental and theoretic results II



(b) Efficiency vs normalized load power

Conclusions

- In this paper an analytic expression was developed and used for quantifying the power-efficiency trade-off involved in DC voltage source that supplies energy to an electric load.

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Conclusions

- In this paper an analytic expression was developed and used for quantifying the power-efficiency trade-off involved in DC voltage source that supplies energy to an electric load.
- The expression was validated by an experimental demonstrator, with good agreement between the results and the predicted values.
- The analytical work developed would be a useful tool for designers and students in order to understand and solve a lot of confusion and myths about power supply efficiency and life time of the battery.

References I

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