Revisiting the Power-Efficiency Trade-Off on a DC Voltage Source

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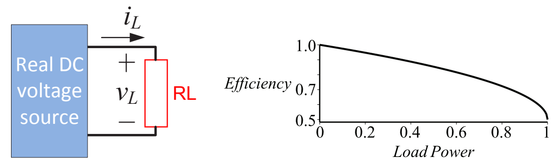
***Abstract*—When a DC voltage source supplies energy to an electric load, it can operate either for maximum power transfer and low efficiency (50%), or with an acceptable efficiency with less power transferred. In this paper an analytic expression is developed and used to quantify this trade off. Moreover, this expression is validated by an experimental circuit demonstrator.**

***Keywords*—Power,Efficiency,Maximum power transfer theorem.**



(a) Analyzed system (b) PETO

Fig. 1. Analyzed energy flow problem.

I. Introduction

In the generic energy flow problem illustrated in the Fig 1(a), the DC source cannot operate at maximum efficiency with maximum transfer power.Therefore, in order to increase its efficiency less power than the maximum available needs to be supplied [1], as illustrated in the Fig. 1(b). This power-efficiency trade-off (PETO) is present in many circuits and systems, such as portable electronic devices and electric motors.

In one of earliest works about PETO (1978), Edison and his chief assistants researched the design techniques for DC electric generators, and determined that a generator with smaller internal resistance than its load is more efficient than a generator with internal resistance equal to its load [2]. Until that moment, the academics believed that the maximum possible efficiency (for any electric energy flow problem) was 50%, reflecting a misunderstanding of the maximum power transfer theorem (MPTT). In [3], an analytical study of the application of the MTTP for AC and DC voltage sources was made and even though it is still at an early development stage, it contains interesting ideas. For instance, the authors of this work propose an indirect analytic expression to quantify the PETO on the DC and AC Thevenin equivalent circuit, using an intermediate variable.

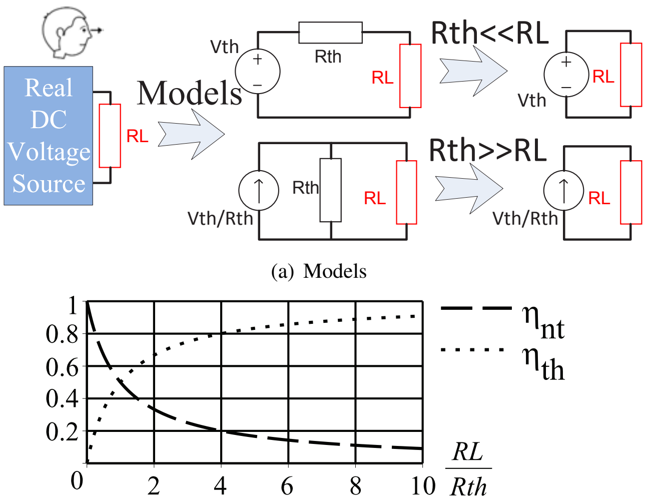
Currently, most of the electronic devices must be portable and must be connected to the Internet [4]. Typically, the device portability is associated to the energy technologies that has been adopted (i.e. battery and DC/DC converters) because they are the least likely element to change during the development cycle of the product [5]. Currently, there are mainly devices powered by batteries [6] and more recently by energy harvesters [7], [8]. However, the design of such devices must face a trade-off between functionality (i.e. dissipated power) and portability (i.e. battery running time, weight, size) [9]. In other words, the engineers need to take into account the PETO in order to achieve the required specifications and, as a result, an optimum operating point (i.e. optimum load) could be found and then is used as a constraint in the electronic design of the system.

Inside the electric circuits textbooks the PETO is not presented in an integrative approach. Typically, these books introduce a discussion of each involving topic, without linking them. For example, in order to calculate the optimum load, the only solved optimization process presented is the MPTT theorem. Probably, for that reason many junior engineers still believe that MTTP solves any energy flow problem [10]. On the other hand many experienced engineers know, the MTTP usefulness is limited in practice because, as it only achieves one optimization criterion (i.e. maximum load power). Likewise, using the conventional circuits theory, the PETO cannot be analyzed directly when the energy source are batteries. This is because the source models proposed by the conventional circuit theory do not represent neither the storage energy capacity of batteries, nor the energy extract process. In [6], a circuit model that captures the charge extraction process in batteries was proposed. This work was extended in [11], where they propose a DC voltage source with limited energy storage capacity in order to model some behaviors of the battery.

This paper found a direct analytic expression (i.e. the source efficiency expressed as a function of the normalized load power) for quantifying the PETO on any source that can be modeled using the circuits shown in Fig.3. This analytic expression allows the design engineer to choose an acceptable operating point of source efficiency ($50\% < \eta < 100\%$) and load power (lower than the maximum available power of the source) in order to design the electronic load.

II. Efficiency in a voltage source

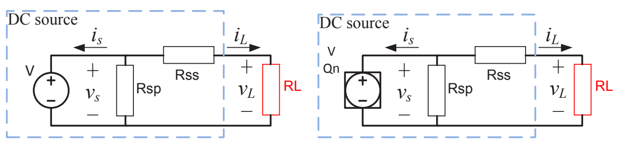
The electrical circuit models use a combination of the circuit elements (i.e. voltage sources, resistors, capacitors,etc.) for modeling the physical reality of the systems. The model complexity is determined by the dynamics that the model



(a) Models

(b) η predicted by the models

Fig. 2. Norton (nt) and Thevening (th)



(a) With infinite energy

(b) With finite energy

Fig. 3. Real DC voltage source models

must predict. This choice defines the interest variables in the modeling problem. Considering a real DC voltage source that supplies energy to a electrical load (1(a)), if we define as interest behavior only the electric load dynamics, the model must predict at least the current and voltage of the load.

A common technique for modeling a real voltage source is the use of the Norton or Thevenin equivalent circuit as its model (Fig.2(a)). These equivalent circuits predict only the current and voltage performance of the source from the load point of view. Therefore, when this modeling technique was used to predict the real sources dynamics a huge error was added to the results. For example, the current and voltage of the load predicted by the Thevenin model are equal to the values predicted by the Norton model, but the efficiency values predicted by both models are different, as is shown in the Fig. 2(b), only when the maximum power transfer condition is satisfied ($R_L = Rth$) both models predict the same efficiency value. Therefore, without knowledge about the physical laws that govern the internal source behaviors, we cannot predict accurately its efficiency.

In the next section, the PETO is discussed for any energy source that can be modeled using the circuits shown in Fig.3. Considering the Fig.3(b), the circumscribed square of the voltage source implies a limited energy of the source. For instance, in [11] was proposed a battery model based on the circuit presented in Fig. 4, it models both the energy restriction of the batteries and the involved energy transfer process.

III. Power-efficiency trade-off

Considering a real DC voltage source that supplies energy to an electric load, it is impossible to deliver the maximum available power to the load and simultaneously to achieve the maximum source efficiency. Further, when the maximum efficiency is achieved the power dissipated by the load is less than the maximum available power of the source ($Pavs$). Furthermore, between these two extreme points, we can only increase the load power by decreasing the efficiency and vice versa. Considering the Fig. 2, the general expression for the energy consumed by the load (E_c), the energy generated by the source (E_g), and the efficiency (η_S) are given by:

$$E_g = t_s \bar{P}_s = -\int_o^{t_s} (i_s(t) \cdot v_s(t)) \cdot dt; \quad (1)$$

$$E_c = t_s \bar{P}_L = \int_o^{t_s} (i_L(t) \cdot v_L(t)) \cdot dt; \quad (2)$$

$$\eta_s = \frac{E_L}{E_s} = \frac{\bar{P}_L}{\bar{P}_s}; \quad (3)$$

where, $\bar{P}_L$ is the mean dissipated power by the load, $\bar{P}_S$ is the mean generated power by the source, and t_S is the runtime of the source. For simplicity in the presentation of concepts, the PETO will be analyzed for the source lossless ($R_{ss} = 0, R_{sp} = \infty$), the source without loss of self consumption ($R_{sp} = \infty$), and finally the general case.

A. *PETO for lossless voltage source* ($R_{ss} = 0, R_{sp} = \infty$)

Using (1), (2) and (3), it was obtained the particular expression for the mean dissipated power and the efficiency:

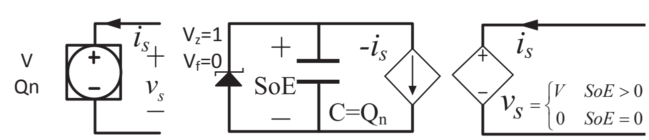
$$\bar{P}_{L_a} = \frac{V^2}{RL}; \quad (4)$$

$$\eta_{s_a} = 1. \quad (5)$$

For this circuit the efficiency is 1 and the maximum available power is infinite. Additionally, the efficiency is a load independent variable, in consequence the PETO does not occur. The efficiency and the dissipated power with respect to resistive load value are shown in the Fig.5(a). Further, the efficiency versus the load power is shown in the Fig.5(b).

B. *PETO for voltage source w/o loss of self consumption* ($R_{ss} \neq 0, R_{sp} = \infty$)

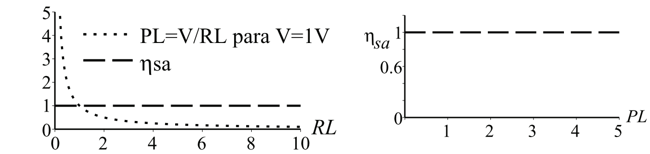
Using (1), (2) and (3), it was obtained the particular expression for the mean dissipated power (6) and the efficiency (7). When $RL = R_{ss}$, the dissipated power by the load is maximum (P_{avs}), this power can be calculated by (8). Using



(a) Symbol

(b) Circuit model

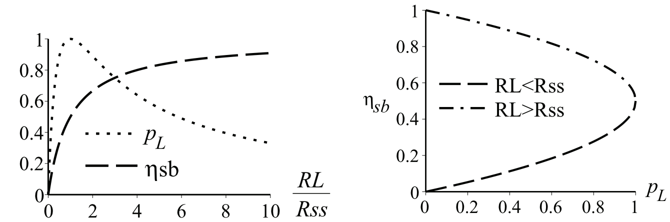
Fig. 4. Ideal battery



(a) Load power and efficiency vs load

(b) Efficiency vs power

Fig. 5. PETO for lossless source



(a) Load power and efficiency vs load

(b) Efficiency vs power

Fig. 6. PETO for a source w/o loss of self consumption

(6) and (7), we obtain the normalized load power (p_L), given by (9).

$$\bar{P}_{L_b} = 4P_{avs}\frac{\frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}; \tag{6}$$

$$\eta_{s_b} = \frac{\eta_{s_a}}{1+\frac{Rss}{RL}}; \tag{7}$$

$$P_{_{avs}} = \frac{V^2}{4\cdot Rss}; \tag{8}$$

$$p_L = \frac{\bar{P}_{L_b}}{P_{_{avs}}} = \frac{4\cdot\frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}. \tag{9}$$

In order to analyze the supplied power and the efficiency of this circuit, we plot them with respect to the normalized load (RL/Rss) in the Fig. 6(a). Considering this figure, it is clear that the efficiency is maximized when the load is much greater than the series resistance of the source ($RL \gg Rss$). However, satisfying this condition implies a reduction of the power delivered to the load. On the other hand, the load power is maximized when the resistive load is equal to the series resistance of the source ($RL = Rs$), which leads to an efficiency of 50%. In order to illustrate this PETO, we plot the efficiency versus power in the Fig. 6(b). When the resistive load is greater than the series resistance of the source ($RL > Rss$), it is clear that the trade-off occurs because we can exchange power for efficiency, and vice versa. The load range where the PETO exists, typically, is the load region where we achieve the better compromise between efficiency and power consumption. For this PETO, the limits are: 100% of efficiency and 100% of p_L. We cannot achieve these limits simultaneously, we can achieve: a p_L of 0% with an efficiency of 100% or a p_L of 100% whit an efficiency of 50%. A good compromise of this PETO is p_L of 75% with an efficiency of 75%. In all the operating points of this PETO, the relationship between the p_L and the efficiency is described by:

$$\eta_{s_b} = 1 - \frac{p_L}{\xi}; \tag{10}$$

where, ξ=2 $\left(1+\sqrt{1-p_L}\right)$, and $RL \geq Rss$.

C. PETO for general voltage source ($R_{ss} \neq 0, R_{sp} \neq \infty$)

Using (1), (2) and (3), it was obtained the particular expression for the mean dissipated power (11) and the efficiency (7). Using (11) and (12), we obtain the normalized load power (p_L), given by (9).

$$\bar{P}_{L_c} = \bar{P}_{L_b}; \tag{11}$$

$$\eta_{s_c} = \frac{\eta_{s_b}}{\left(\frac{\mathrm{Rss}}{\mathrm{Rsp}}\left(\frac{\mathrm{RL}}{\mathrm{Rss}}+1\right)+1\right)}; \tag{12}$$

$$p_L = \frac{\bar{P}_{L_c}}{P_{_{avs}}} = \frac{4\cdot\frac{\mathrm{RL}}{\mathrm{Rss}}}{\left(1+\frac{\mathrm{RL}}{\mathrm{Rss}}\right)^2}; \tag{13}$$

In order to analyze the supplied power and the efficiency of this circuit, we plot both with respect to the normalized load (RL/Rss) in the Fig. 7(a). From the graph, it is clear that the efficiency and the load power are maximized in a different load value. The P_{avs} is supplied when (14) is respected. On the other hand, the maximum efficiency is achieved when (15) is satisfied, and its value is given by (16). It is important to notice that RL_p is always greater than or equal to RL_η.

$$RL = RL_p = R_{ss}; \tag{14}$$

$$RL = RL_\eta = Rss\ \sqrt{k+1}; \tag{15}$$

$$\max\{\eta_{s_c}\} = \frac{k\sqrt{k+1}}{\left(1+k+\sqrt{k+1}\right)\left(k+\sqrt{k+1}\right)}; \tag{16}$$

where, RL_p is the load value that maximizes the consumed power. RL_η is the load value that maximizes the efficiency, k is a qualitative measure of the source quality given by $k = \frac{Rsp}{Rss}$. Having a high k value means that the power supply is good, i.e. both the self consumption and the internal conduction losses are low.

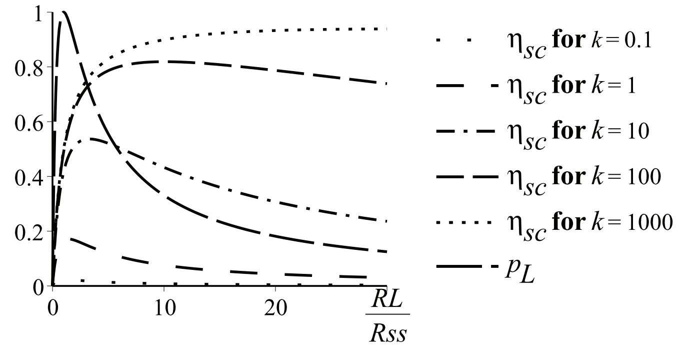
When RL is in the PETO range ($RL_p \leq RL \leq RL_\eta$), the load power can increase only if the efficiency decrease. In order to illustrate this, the efficiency versus power graph is plotted in the Fig. 7(b). In the extreme points of PETO, we can achieve the highest p_L with the lowest efficiency, or vice versa. These limits can be calculated as a function of the k value, and are plotted in Fig. 8. In this source, the relationship between p_L and the efficiency is described by:

$$\eta_{s_c} = \frac{1-\frac{p_L}{\xi}}{1+\frac{\xi}{k\cdot p_L}} \tag{17}$$

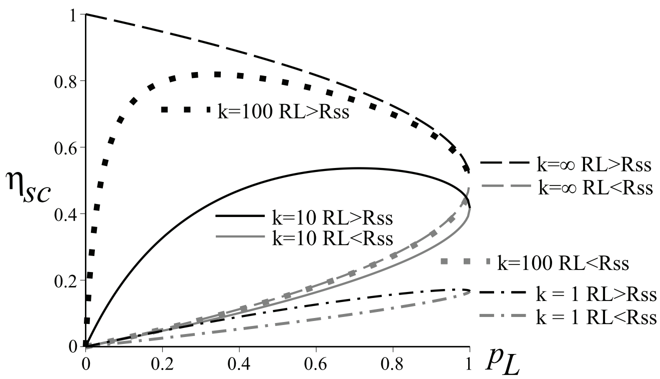
where, $Rss\sqrt{k+1} \geq RL \geq Rss$ and ξ=2 $\left(1+\sqrt{1-p_L}\right)$.

IV. Experimental verification

In this section the analytic function proposed, equation (17) was tested with a experimental setup. The implemented system is a demonstrator of the validity of the analytic function rather than targeting a specific application. Considering Fig.1(a), the real DC source ($k = 90$) was emulated using a power supply (1 V $< V <$ 10 V) and two discrete resistors ($Rsp = 100$ kΩ, $Rss = 1.1$ kΩ), the electric load was an adjustable resistor (100mΩ $< R_L <$ 10 MΩ). Using this setup, both the resistive load and the input voltage were swept. The PETO results were plotted in the Fig. 9. Considering this figure, there is a good agreement between the experimental results and the values predicted by the expression (17).



(a) Load power and efficiency vs load



(b) Efficiency vs power

Fig. 7. PETO for the voltage source

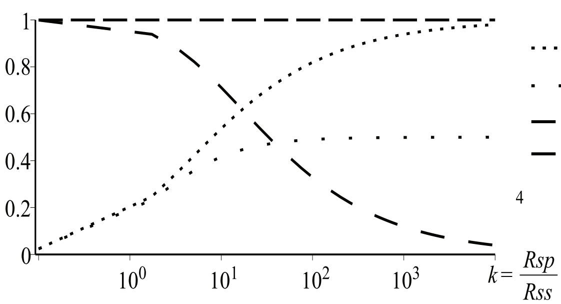


Fig. 8. Efficiency and load power boundaries vs k

V. Conclusions

In this paper an analytic expression was developed and used for quantifying the power-efficiency trade-off involved in DC voltage source that supplies energy to an electric load. Further, this expression was validated by an experimental demonstrator, with good agreement between the results and the predicted values. Furthermore, this analytical work would be a useful tool for designers and students in order to understand and solve a lot of confusion and myths about power supply efficiency and life time of the battery.

Acknowledgment

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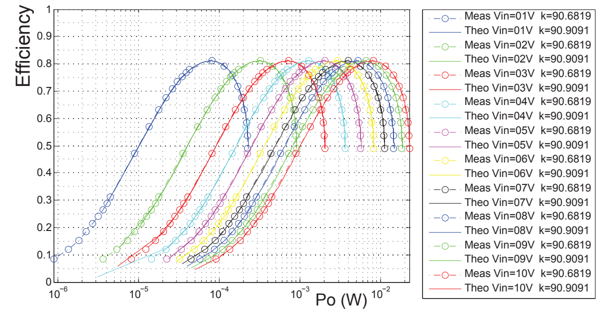
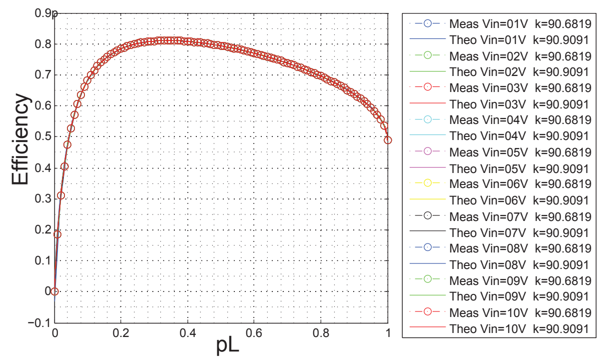


(a) Efficiency vs load power



(b) Efficiency vs normalized load power

Fig. 9. Experimental and theoretic results for PETO on a emulated DC voltage source of $k = 90$

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